Kinematics

By the end of this unit you will be able to:

- **Define displacement, velocity, and acceleration**
- **Use the definition of velocity to solve word problems**
- **Use the four kinematics equations to solve word problems in the horizontal plane**
- **Use the four kinematics equations to solve word problems in the vertical plane**

- **Linear Motion**
  - Motion in one dimension
    - This dimension could be horizontal or it could be vertical
      - An object thrown up in the air undergoes linear motion
      - So does a car driving in a straight line
  - Distance (x)
    - Distance is measured in meters (m)
  - Time (t)
    - Time is measured in seconds (s)
  - Speed (v)
    - Units: m/s
    - Defined to be the distance covered in the elapsed time

$$v = \frac{x}{t}$$

- Example
  - The distance from Dayton to Columbus is 114,424 m. If it takes you 71.1 minutes to drive from one to the other, what is your speed?

x = 114,424 m

t = 71.1 min.

t = 4266 s

$$v = \frac{x}{\Delta t} \rightarrow v = \frac{114424m}{4266s} \rightarrow v = 26.8\ \mathrm{m/s}$$

- When driving the distance above, are you always going 26.8 m/s?
- The speed found is the average speed
- Average speed is the distance covered over the change in time

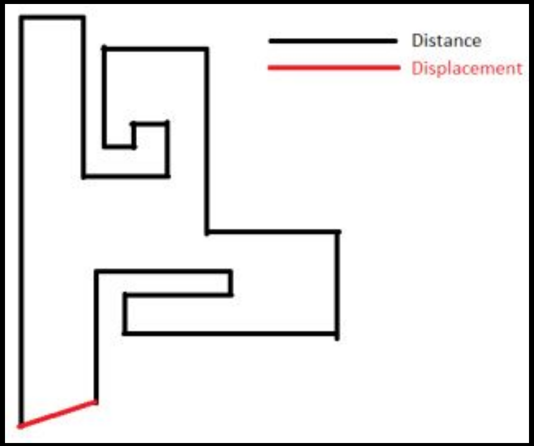
$$v = \frac{x}{\Delta t}$$

Δ stands for “change in”

- Displacement (x)
  - Vector form of distance
    - Units: meters
  - Magnitude of the displacement is the straight line distance from the initial to final positions
  - Vector points from the initial to final positions

$$\vec{x} = x_f - x_o$$

x_f and x_o are the final and initial positions



In the picture, the distance traveled in in black and the displacement is in red.

Can you ever travel a distance and end up with a displacement = 0?

- Velocity (v)
  - Vector form of speed
    - Units: meters per second
  - The magnitude of the velocity is the speed of the object
    - In other words, the rate that you are changing your position
      - That is why the units are distance per time
  - Direction of the velocity is the direction of the motion

$$\vec{v} = \frac{x}{\Delta t}$$

Δt = "change in time"

From here on in, whenever you see a "t" for time it really stands for "Δt".

- Instantaneous Velocity
  - The average velocity is how fast you were going the entire trip
    - Requires simple algebra to calculate
  - The instantaneous velocity is how fast you are going at that moment
    - Requires calculus to calculate
    - Unfortunately, we will not be calculating the instantaneous velocity of any objects in this class
    - Instantaneous velocities are not easy to calculate, but they are easy to measure. What would you use to measure it?

$$\vec{v}_{\text{Instant}} = \lim_{t \to 0} \frac{\vec{x}}{t}$$

- Acceleration (a)
  - Defined to be a change in velocity
    - Units = meters per second per second
      - m/s^2
    - Change in velocity is a change in speed or a change in direction or both
    - That means that accelerations occur when you change your speed or direction or both
      - Change in speed means speeding up or slowing down
        - » The word deceleration means “negative acceleration”
  - The magnitude of the acceleration is the rate that the speed is changing
    - That is why the units are speed per time

- The acceleration vector points in the direction of motion if the object is speeding up
  - This vector is positive
- The acceleration vector points in the opposite direction of motion if the object is slowing down
  - This vector is negative

– Average acceleration is your change in speed over the elapsed time

$$\vec{a} = \frac{\Delta \vec{v}}{t}$$

– Instantaneous Acceleration

- How much you are accelerating at that moment
  - Once again, this requires calculus to calculate
  - We won't be calculating instantaneous accelerations either in this course

$$\vec{a}_{\text{Instant}} = \lim_{t \to 0} \frac{\vec{v}}{t}$$

- Kinematics Equations
  - These equations are used to describe motion under a constant acceleration
    - Constant acceleration means that the acceleration is not changing
  - There are four kinematics equations
    - We will use these four equations to help us start to describe motion
  - There are five kinematics variables
    - x = displacement
    - v_o = initial velocity
    - v = final velocity
    - a = acceleration
    - t = elapsed time

Start with the definition of acceleration

$$a = \frac{\Delta v}{t}$$

Since "Δ" stands for "Change in",
substitute the change in velocity for "Δv"

$$a = \frac{v - v_o}{t}$$

Solve the above equation for the final velocity

$$v = v_o + at$$

Start with the definition of average velocity

$$v = \frac{x}{t}$$

Since the average velocity can also be found by averaging the final and initial velocity, substitute this for “v”

$$\frac{1}{2}\left(v + v_o\right) = \frac{x}{t}$$

Solve the above equation for distance

$$x = \frac{1}{2}\left(v_o + v\right)t$$

Take the first equation we derived and substitute it in for "v" in the second equation

$$v = v_o + at$$

$$x = \frac{1}{2}(v_o + v)t$$

$$x = \frac{1}{2}(v_o + (v_o + at))t$$

Simplify the above equation

$$x = v_o t + \frac{1}{2}at^2$$

Solve the first equation we derived for "t"

$$v = v_o + at$$

Substitute this equation into the second one we derived

$$t = \frac{v - v_o}{a}$$

$$x = \frac{1}{2}(v_o + v)t$$

$$x = \frac{1}{2}(v_o + v)\left(\frac{v - v_o}{a}\right)$$

Simplify the above equation and then solve for "v^2"

$$v^2 = v_o{}^2 + 2ax$$

– Summary of Kinematics Equations

$$v = v_o + at$$

$$x = \frac{1}{2}(v_o + v)t$$

$$x = v_o t + \frac{1}{2}at^2$$

$$v^2 = v_o{}^2 + 2ax$$

While I will never have you memorize equations, it would be worth your while to learn these. We will be using them **ALL** year.

- Free Fall
  - Free Fall occurs when gravity is the only influence on an object
  - Any falling object is under a constant acceleration
    - Since it is under a constant acceleration the kinematics equations apply
  - The acceleration due to gravity (g) is 9.8 m/s^2 near the surface of the Earth
    - If you get "far" away from the surface of the Earth this value will change
    - You need to memorize this number

$$g = 9.8 m / s^2$$

- The acceleration due to gravity vector points towards the surface of the object
  - Because gravity always accelerates objects towards the center of the object

– What is the velocity of a falling object dependent on?

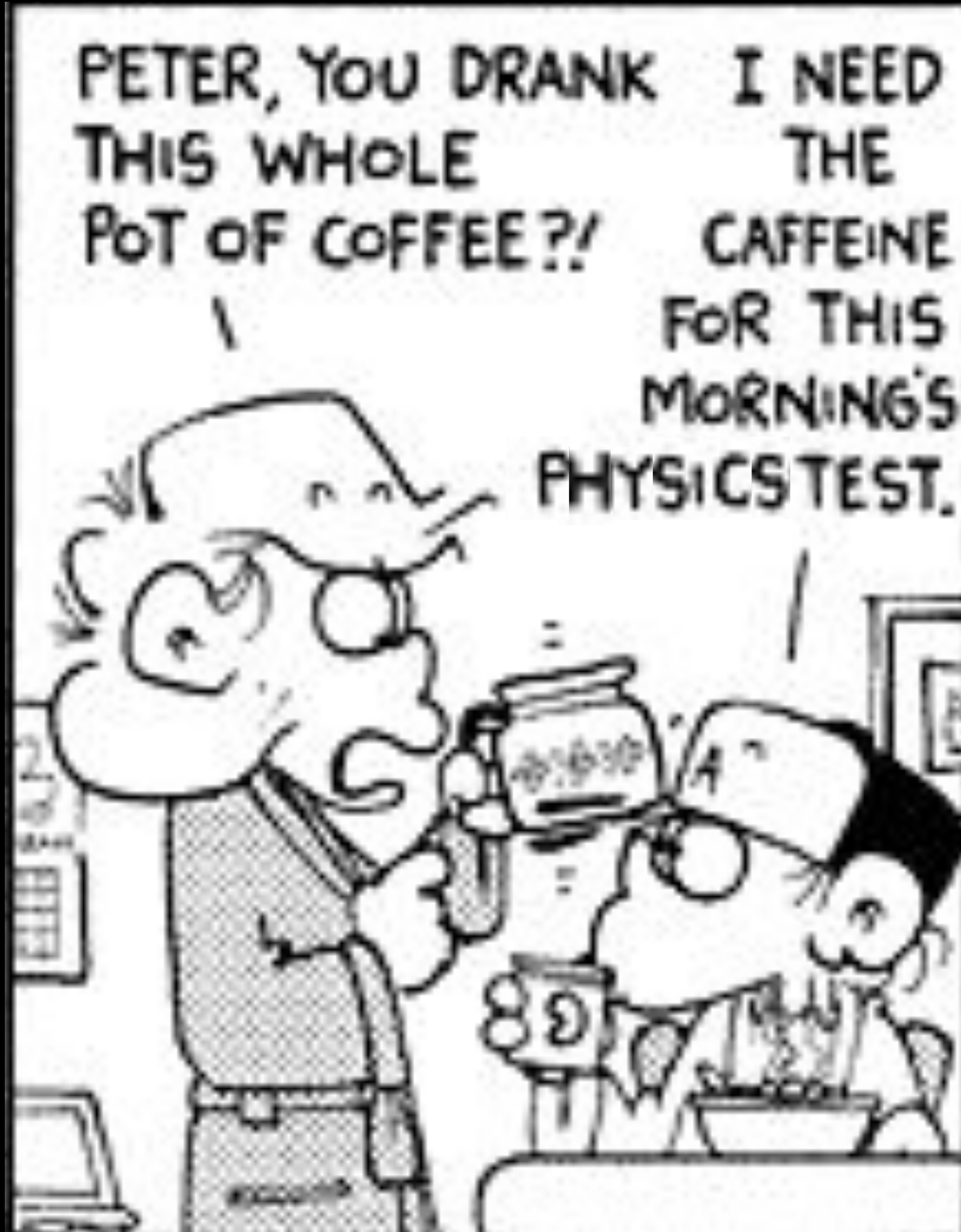
- To find out, go the first kinematic equation and plug in “g” for the acceleration

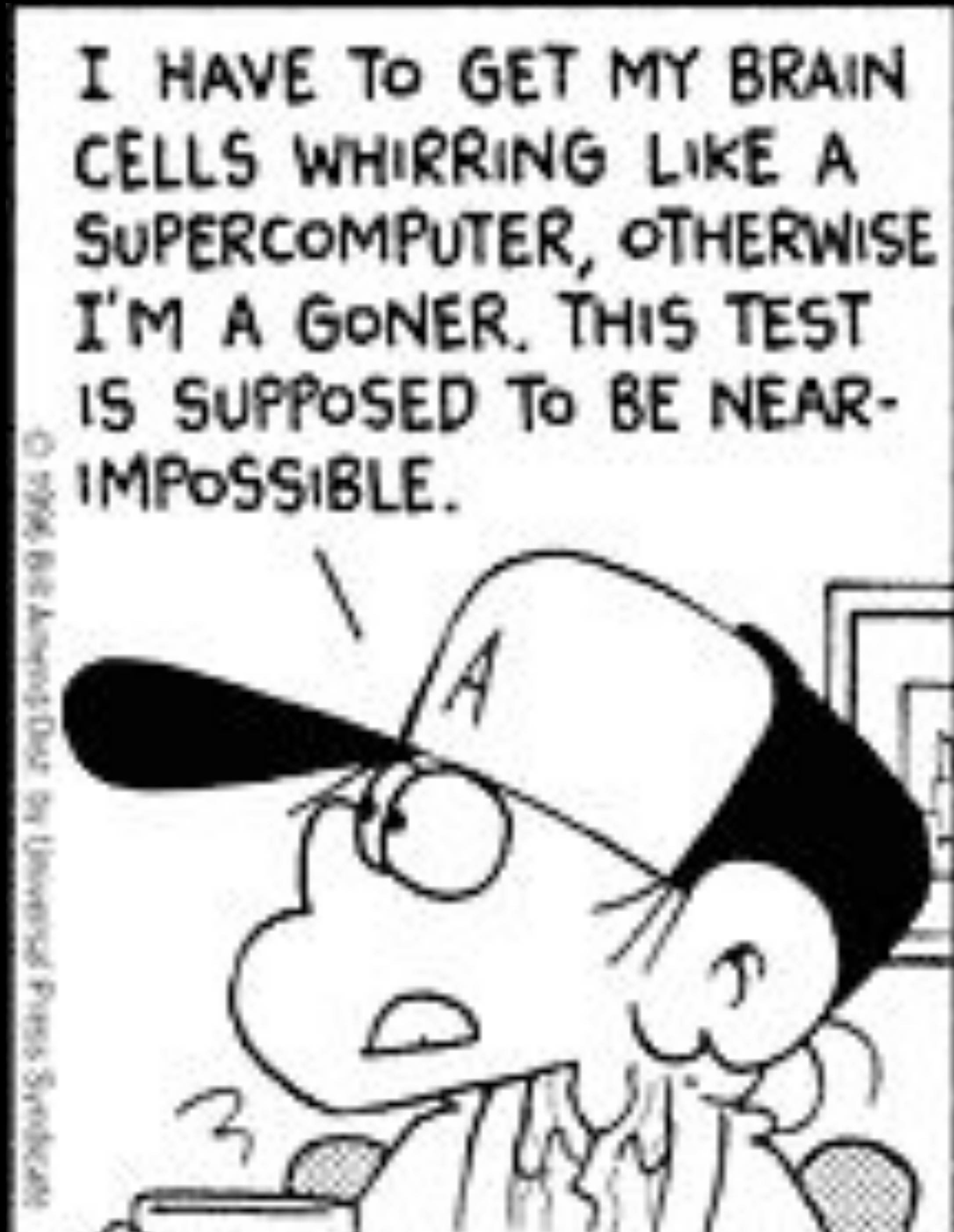
$$v = v_o + gt$$

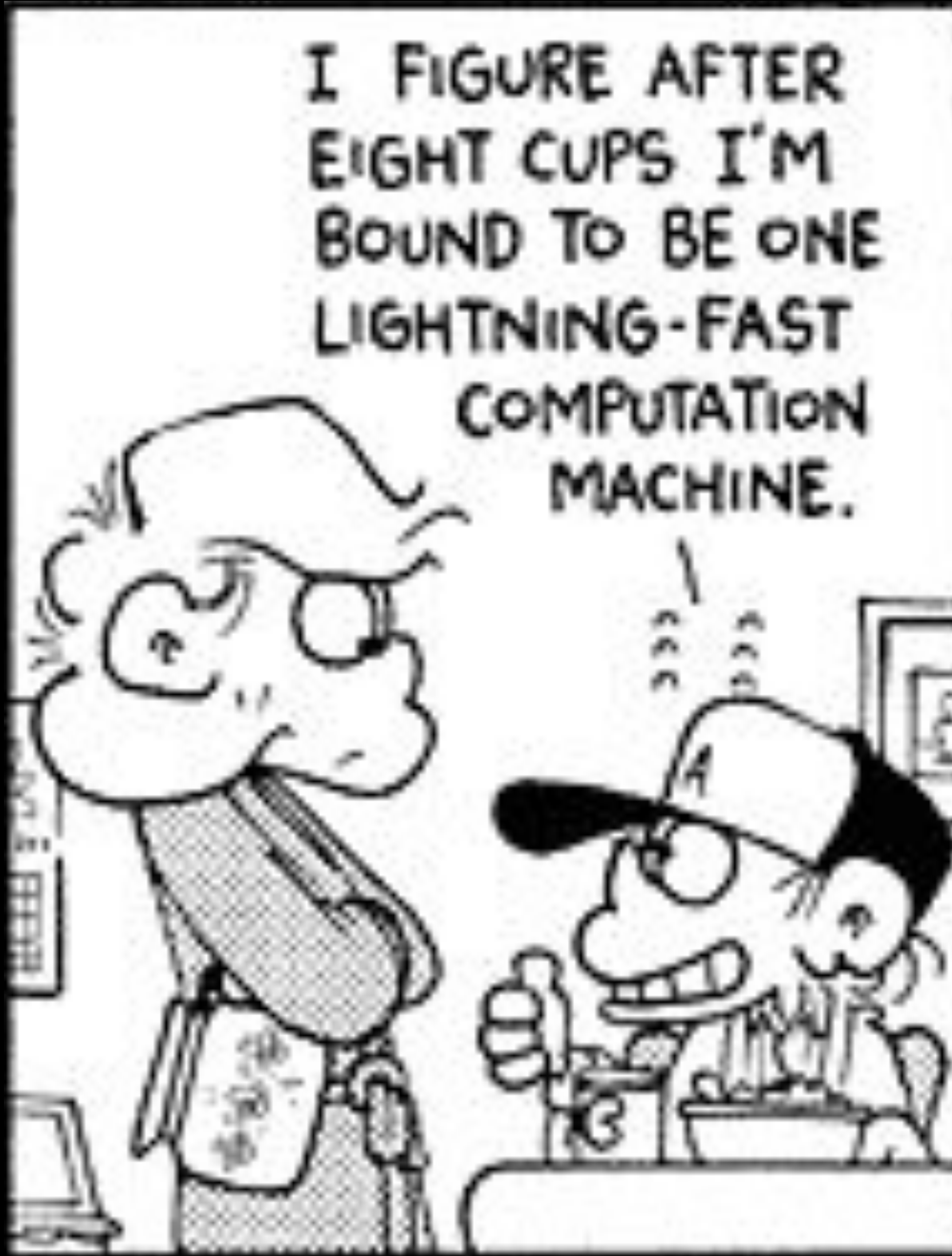
- The velocity of a falling object on Earth is dependent on the time it has fallen and the initial velocity
  - That’s it and nothing more
  - Notice that the velocity of an object does not depend on the mass
    - » In other words, objects of different mass fall at the same rate.
  - We can test this on the Earth, but to get an accurate test, we must go to the Moon
    - » This is because the Moon has no air and no air resistance

Astronaut David Scott of Apollo 15

He is testing whether objects fall at the same rate in the absence of an atmosphere


PETER, YOU DRANK THIS WHOLE POT OF COFFEE?!
I NEED THE CAFFEINE FOR THIS MORNING'S PHYSICS TEST.


I HAVE TO GET MY BRAIN CELLS WHIRRING LIKE A SUPERCOMPUTER, OTHERWISE I'M A GONER. THIS TEST IS SUPPOSED TO BE NEAR-IMPOSSIBLE.


I FIGURE AFTER EIGHT CUPS I'M BOUND TO BE ONE LIGHTNING-FAST COMPUTATION MACHINE.


IF THE BOYS' BATHROOM IS 150 FEET AWAY AND I RUN AT A RATE OF 20 FEET PER SECOND...
MR. FOX, IS THERE A REASON YOU KEEP SQUIRMING?
AMEND 1-11