Energy

By the end of this unit, you will be able to:

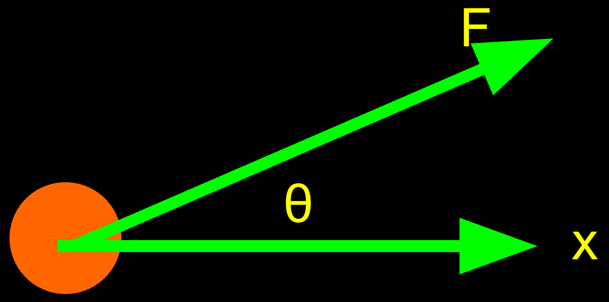
- Define and calculate work done by a constant and variable force
- Define and calculate power exerted
- Define and calculate kinetic energy
- Perform calculations with the work-energy theorem
- Describe conservative and non-conservative forces
- Define and calculate potential energy
- Describe the Law of Conservation of Energy
- Describe the Law of Conservation of Mechanical Energy
- Perform calculations with the Law of Conservation of Mechanical Energy

Work

- Work (W)
  - Work is defined as the ability to move an object
    - Units: Newton·meter, also known as the Joule (J)
    - Work has two elements: Force and Distance
      - When you move an object it requires a force
      - When you move an object it moves a certain distance
    - Since it does not matter the direction that you move an object, just the fact that you do move it, work is a scalar quantity
    - In fact, work is mathematically defined to be the product of the force exerted on an object and the distance it moves

$$W = Fx$$

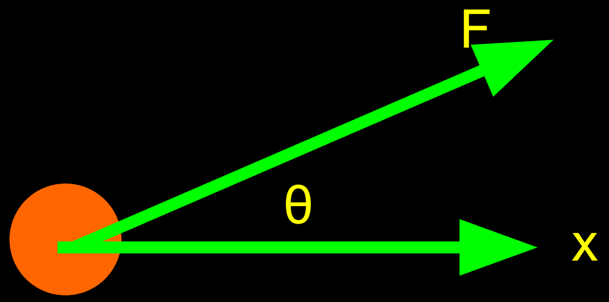
- The above equation only works if the force is in the same direction as the displacement
- If the force is at an angle relative to the the displacement, then we need to know how much force is in the direction of the displacement
  - In other words, we need to know the component of the force in the direction of the displacement



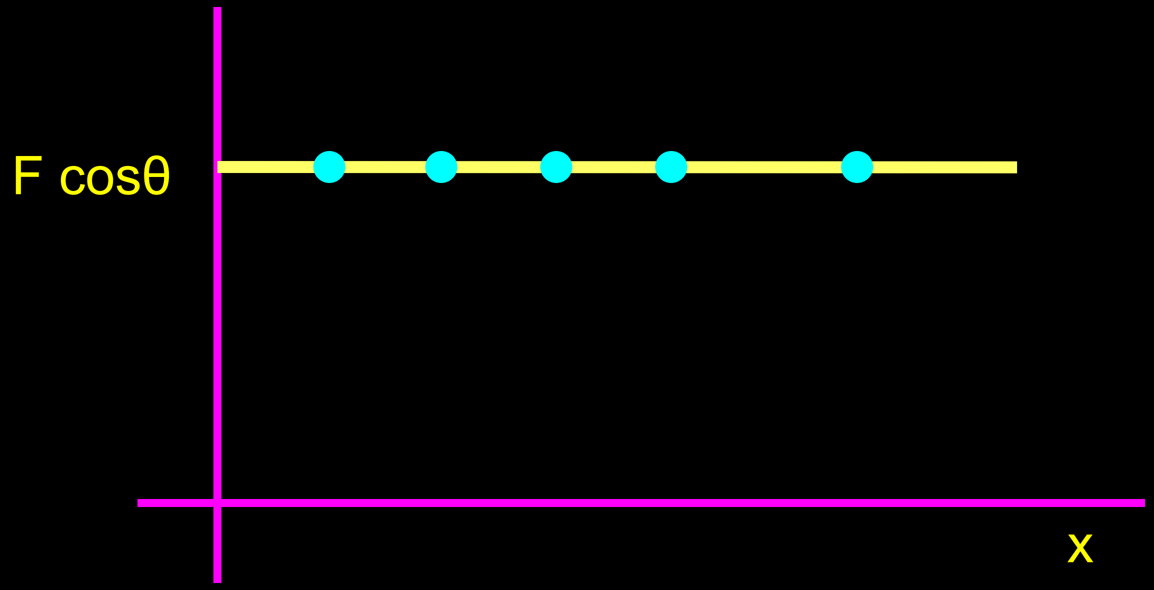
- If it is assumed (rightfully so) that the force is always at some angle from the displacement (even if that angle is zero) then the work equation turns into

$$\boxed{W = Fx\cos\theta}$$

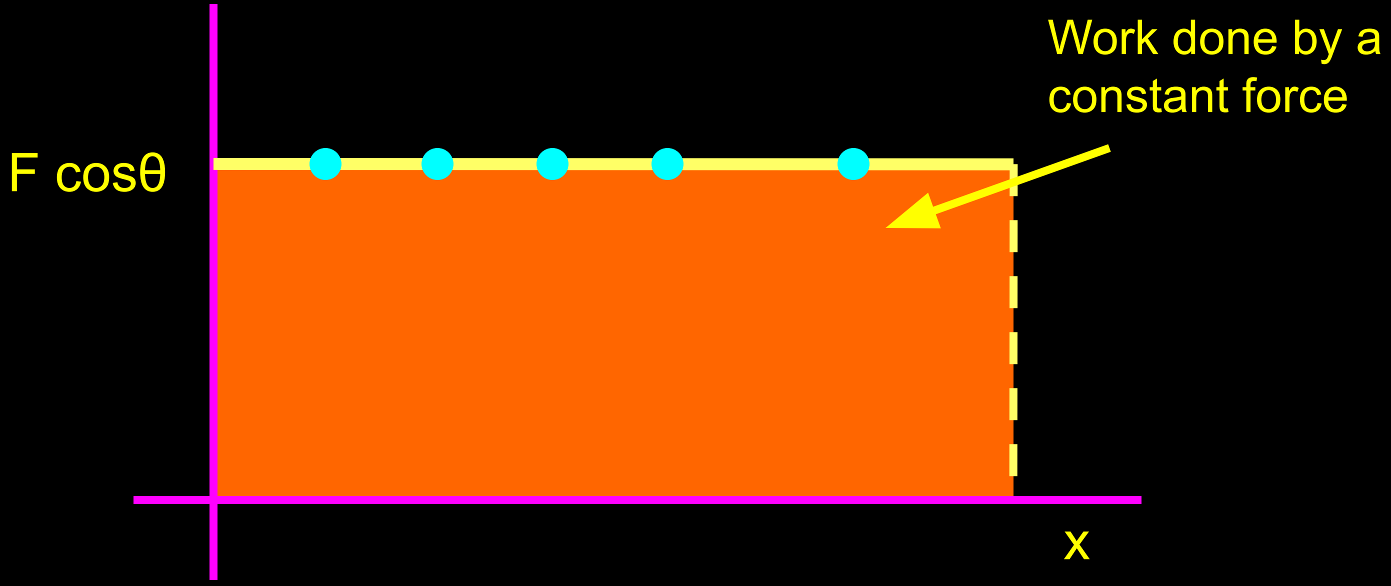
where θ is the angle between the force and the displacement



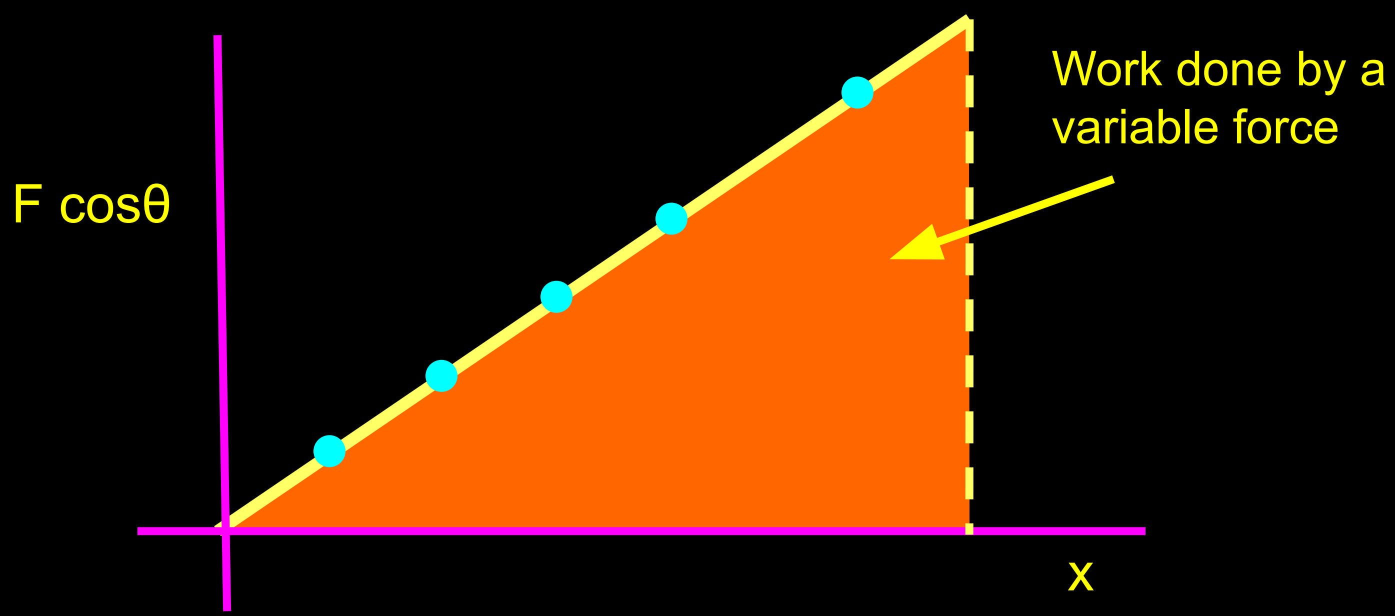
- Work Done by a Variable Force
  - One last consideration before we move on
    - Until now, the force is always been assumed to be a constant force
    - If the force being applied to the object is not constant then our work equation does not hold true
      - Before we get there, let’s look at a graphical representation of a constant force



- From our equation, the work is equal to Fcosθ multiplied by x
- From the graph, this also finds the rectangle that is formed below the graph but above the x-axis
- To generalize, the work done by a constant force is equal to the area of the rectangle formed by the graph x vs. Fcosθ

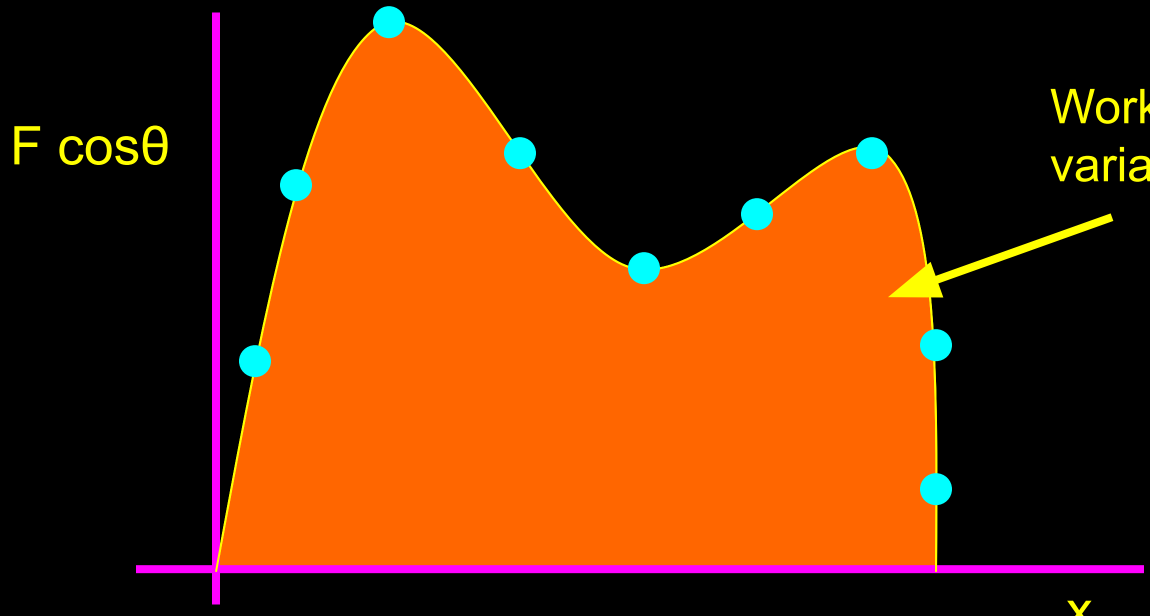


- Does this hold true for forces that are not constant?
  - It turns out it does
  - The area may not be a rectangle, but the principle holds
  - The work done by a force, constant or otherwise, is equal to the area between the graph and the x-axis



  - To find the work done by this force you would simply have to find the area of the triangle

- No matter the shape, the area under the graph is the work done by the force



- For non-basic shapes, there is only one way to calculate the area under the curve
- This way is called the integral

– So, a more general equation to find the work done by a force is as follows

$$W = \int F(x)x\cos\theta dx$$

Power

- Power (P)
  - Units: Joule/second also known as the Watt (W)
    - What is the difference, in terms of physics, between walking up the stairs and running up the same set?
    - Each time you do the same amount of work
      - It is the same object traveling the same distance under the same force
    - However the time it takes you to do the work is different, which leads us to power
  - Power is how much work is done per unit of time

$$\boxed{P = \frac{W}{t}}$$

- Traditionally the unit on power is horsepower
  - This probably stems from early inventors and scientists comparing their engines to a horse, something everyone at the time was familiar with
- 1 horsepower was, at the time, defined as the power exerted by one average horse
  - So if an engine sported 4 horsepower, it could generate the power of 4 horses
    - » Quite the selling point early in the Industrial Revolution
- Nowadays, 1 horsepower is defined as being equal to 746 watts

– Example

- A 1.1 x 10^3 kg car, starting from rest, accelerates for 5 s. The magnitude of the acceleration is 1.6 m/s^2. Determine the average power generated by the net force that accelerates the vehicle under the assumption that the force and the distance traveled are in the same direction.

$m = 1.1 \times 10^3$ kg

$a = 1.6$ m/s^2

$v_0 = 0$ m/s

$t = 5$ s

$\theta = 0°$

We are looking for power, so let's start with the power equation

$$P = \frac{W}{t}$$

The only thing I know nothing about is work, so let's substitute

$$P = \frac{Fx\cos\theta}{t}$$

From here, everything can be found using previous equations/skills

$$P = \frac{Fx\cos\theta}{t}$$

F = 1760 N

x = 20 m

$$P = 7040\,W$$

Energy

- Energy
  - Energy is defined to be the ability to do work
  - Energy is a scalar quantity and also measured in Joules (J)
    - This makes sense because work is a form of energy
  - Traditionally defined as having two different types: Kinetic and Potential

- Kinetic Energy (K)
  - Kinetic energy is defined as the energy of motion
    - Work, since it is defined as when a force moves an object, is related to kinetic energy for obvious reasons
    - Since kinetic energy is the energy of motion then it should be dependent on the velocity of the object
    - If you have two objects moving at the same velocity, the more massive object has more energy
      - So kinetic energy should also be dependent on mass
  - The kinetic energy of an object is defined as the

$$\boxed{K = \frac{1}{2}mv^2}$$

- Work – Energy Theorem
  - I mentioned on the previous slide that work and kinetic energy have to be related
    - On the next two slides we are going to derive the relationship between work and kinetic energy

Start with Newton's Second Law

$$F = ma$$

Since this is an equality, I can multiply both sides by the same quantity and maintain the equality

So I am going to multiply both sides by distance

$$Fx = max$$

You should recognize the term on the left, so substitute in the correct quantity

$$W = max$$

Now, let’s substitute in for *ax*

$$W = max$$

I’ll give you an equation for *ax* to substitute into the above equality

$$ax = \frac{v^2 - v_o^2}{2}$$

Now, let’s substitute in for *ax*

$$W = m\frac{v^2 - v_o^2}{2}$$

Distribute the mass through the fraction and then split it into two quantities

$$W = \frac{1}{2}mv^2 - \frac{1}{2}mv_o^2$$

Substitute in for the appropriate quantities on the right side of the equality

$$W = K - K_o$$

$$\boxed{W = K - K_o = \Delta K}$$

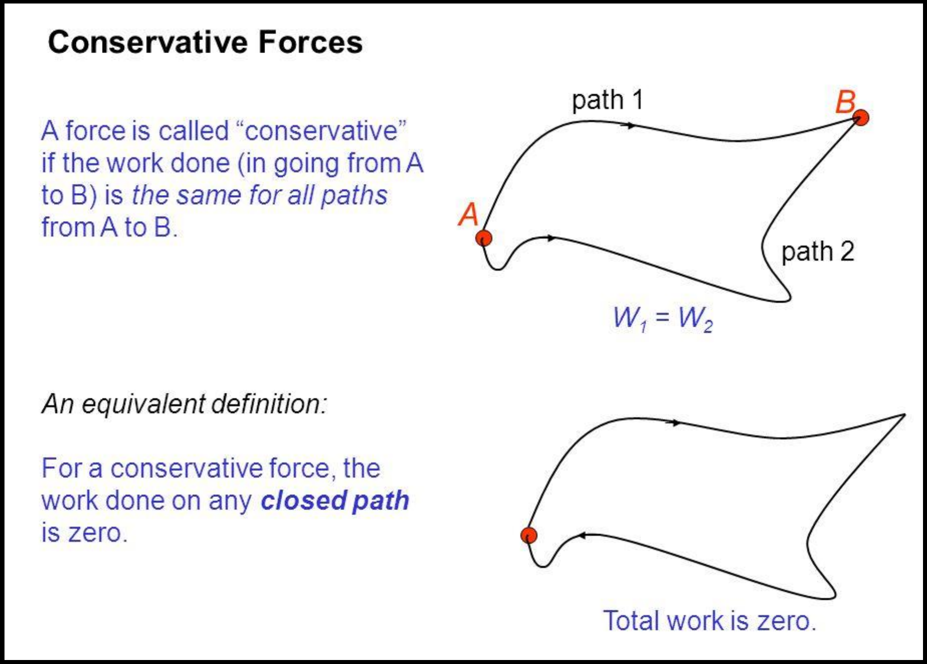
- So the work done on an object is equal to the change in kinetic energy of the object
  - So, if there is zero change in kinetic energy then there is zero work done on the object and vice versa
  - Work through the example below

- Example 1
  - A space probe (m = 5×10^4 kg) is traveling at a speed of 1.1×10^4 m/s through deep space. The engine exerts the only force (4×10^5 N) on the probe. The force is acts in the direction of the displacement. If the probe travels for 2.5×10^6 m, what is its final speed?

    - $v = 1.27 \times 10^4$ m/s

- Potential Energy (U)
  - Potential energy is defined as energy that will do work, as long as the conditions are correct
    - This energy can come in a variety of forms, all that is required is that the energy is not currently doing work
    - Only occurs with conservative forces

- Conservative and Non-Conservative Forces
  - Conservative Force
    - If the work done by a force is independent of the path of motion, then the force is called a conservative force
      - This means that the work done by the force will be the same no matter what path the object takes
    - Gravitational force, electromagnetic force, and elastic force in a spring are all examples of conservative forces
      - These forces have a potential energy associated with them



- Non-conservative Forces
  - A force is non-conservative if the work done by the force is dependent of the path of motion
    - This means that the work done by a force can change depending on the path an object will take
  - Examples of non-conservative forces are friction forces, air resistance, normal force, and propulsion force of a rocket
    - These forces do not have a potential energy associated with them

– Gravitational Potential Energy

- The force of gravity has the potential to do work on an object
- If gravity does work on an object, then it is pulling it closer to the source of the gravity
- Let's derive a mathematical description of gravitational potential energy

Let's start with the definition of work

$$W = Fx\cos\theta$$

The direction of the force of gravity and the direction an object falls when it is released are the same

$$W = Fx$$

The distance the object falls would be the change in height from where it was to where it ended up

$$W = F\left(h - h_o\right)$$

Assuming that h_f and h_o are much smaller than the radius of Earth

The force of gravity is also known as the weight of an object

$$W = mg(h - h_o)$$

So the work done by gravity is equal to the difference in the term *mgh*

$$W = mgh - mgh_o$$

This term *mgh* is defined as gravitational potential energy

$$\boxed{U_G = mgh}$$

Therefore, the work done by gravity is equal to the difference in gravitational potential energy of the object

$$W = U_G - U_{G_o}$$

$$\boxed{U_G = mgh}$$

- The height in the equation is measured from an arbitrary zero point
  - In other words, you will choose the spot in the problem where the height is equal to zero
  - Normally, but not always, it is smart to set the bottom of the problem equal to zero
    - That way you do not have to deal with negative heights
- Gravitational potential energy, like kinetic energy and work, is a scalar and is measured in joules

- Law of Conservation of Energy
  - The amount of energy in the universe is constant
    - This statement, while true, it not very useful
    - The statement on the next page is much more useful when talking about small scale objects

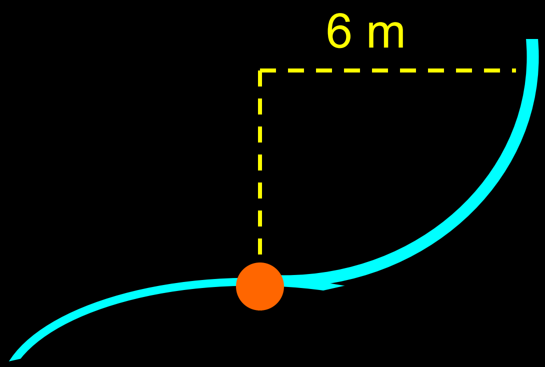
- Law of Conservation of Mechanical Energy
  - For any object, the amount of energy of an object remains constant as the object moves as long as the net work done by non-conservative forces is zero
    - In other words, energy remains constant as long as the forces like friction and air resistance are zero
    - This says nothing about the amount of kinetic or potential energy specifically, just the total amount
    - Therefore, this allows the possibility for energy to be converted from one form to another; from kinetic to potential energy and vice versa

- In equation form, the total energy is equal to the sum of the kinetic energy and the potential energy anywhere along the object's path

$$\boxed{E_T = K_f + U_f = K_o + U_o}$$

– Example (To Be Worked In Class)

- A 6 m rope is tied to a tree limb and used as a swing. A person starts from rest with the rope held in a horizontal orientation. Ignoring friction and air resistance, determine how fast the person is moving at the lowest point on the circular arc of the swing.



- Work Done by Non-conservative Forces
  - When the non-conservative forces acting on an object are not equal to zero, mechanical energy of the object is not conserved
  - This means that the object will have a different amount of energy from the initial state to the final state
  - The difference in energy, or ΔE, is the work that is done by the non-conservative forces
    - Example: A sled sliding down the hill will have less energy at the bottom than the top. This loss of energy is due to the friction from the snow acting on the sled

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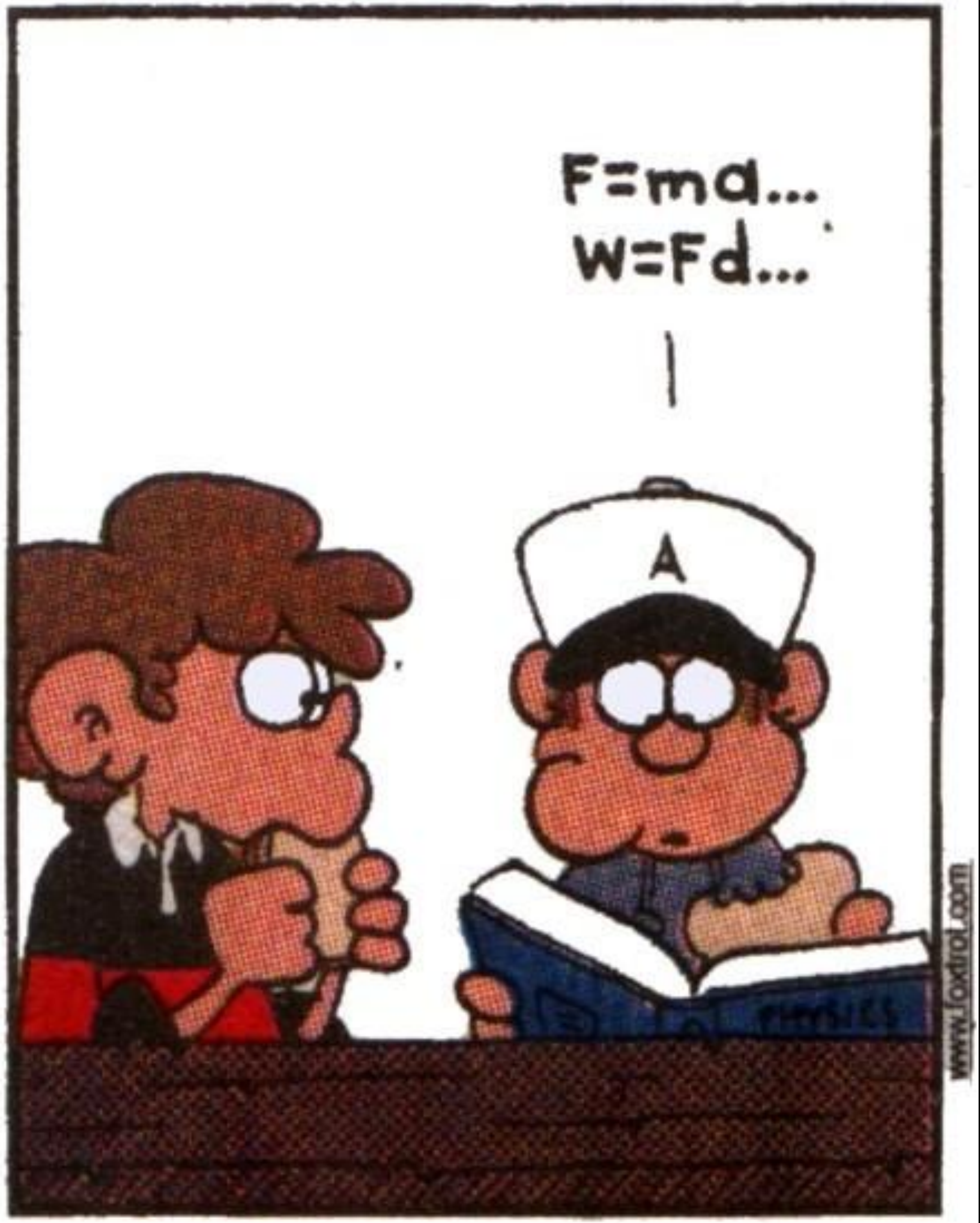

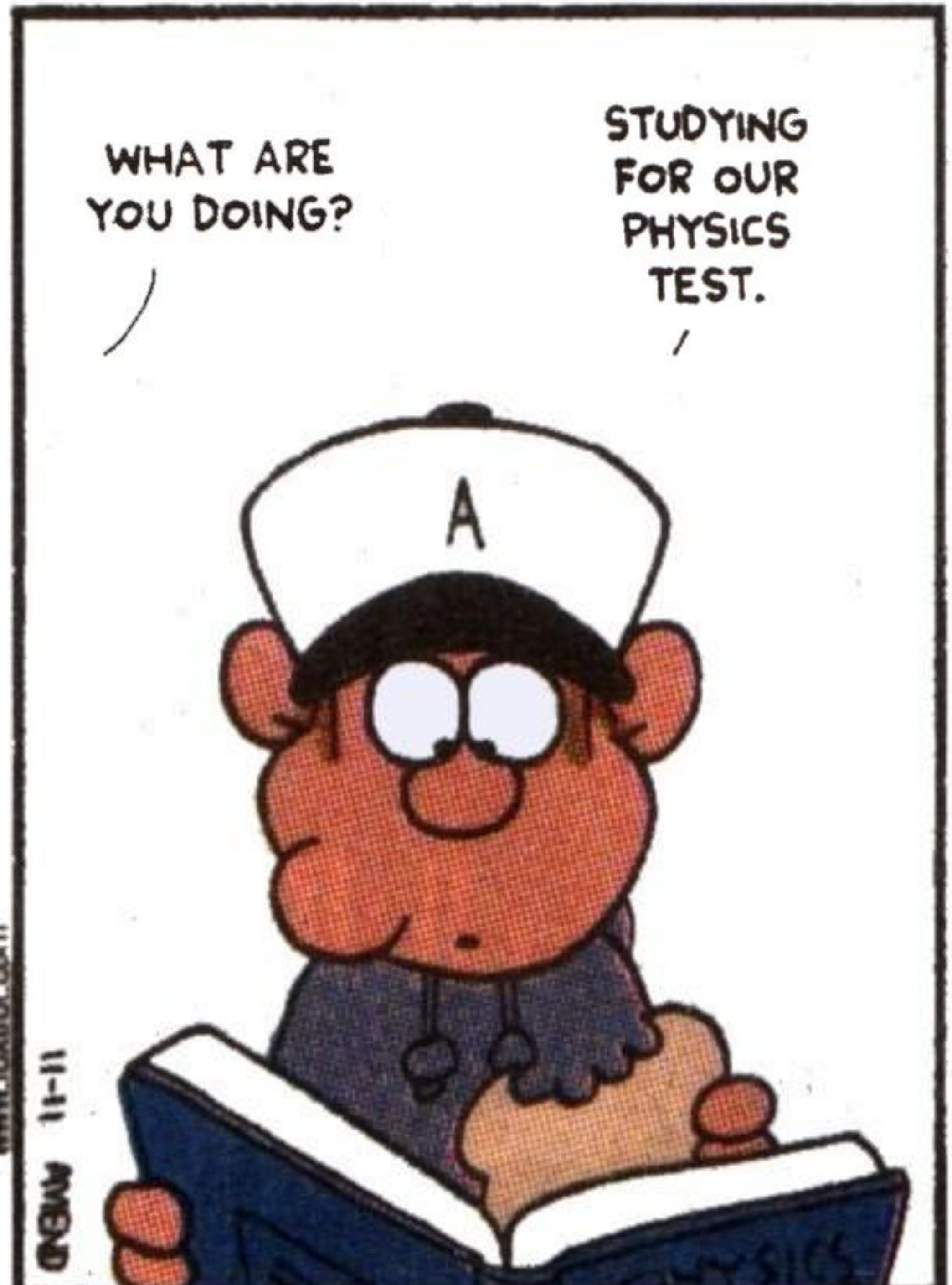
WHAT ARE YOU DOING?
STUDYING FOR OUR PHYSICS TEST.
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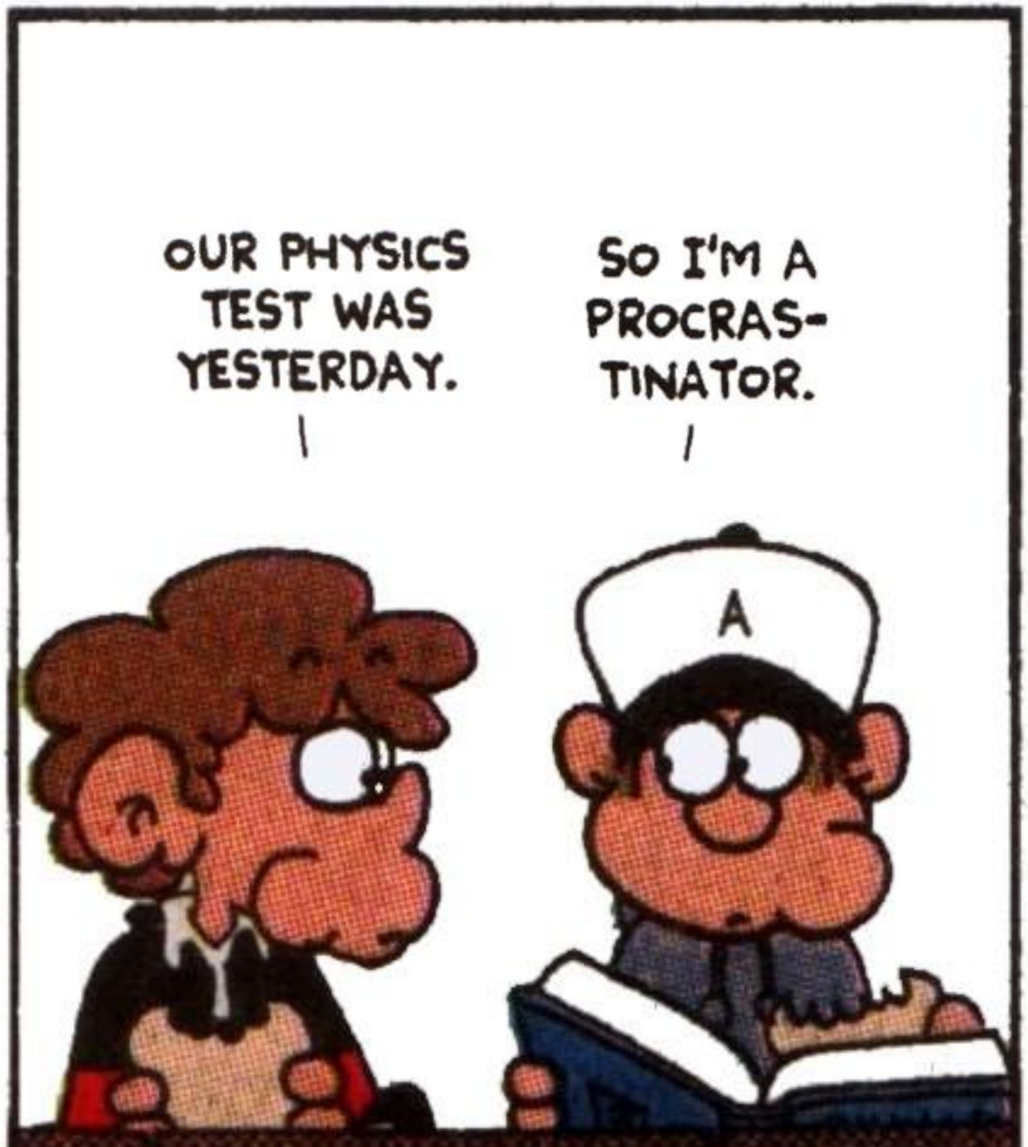
OUR PHYSICS TEST WAS YESTERDAY.
SO I'M A PROCRAS-TINATOR.