# Circular Motion

By the end of this unit, you will be able to:

Define and be able to calculate tangential velocity

Define and explain centripetal and centrifugal forces

Solve problems with rotational kinematics

Be able to convert between angular and tangential terms

Be able to define the direction of a vector using the right-hand rule

Define and perform calculations with torque

Define and perform calculations with moment of inertia

Solve problems with Newton's 2nd Law for Rotation

Solve problems with rotational equilibrium

Solve problems with angular momentum

Solve problems with conservation of angular momentum

Solve problems with rotational work and rotational kinetic energy

Solve problems with conservation of energy involving rotating objects

# Circular Motion

- Circular Motion
  - Circular motion deals with objects that are rotating
  - Any object that is rotating must have a point that it is rotating around
    - This is called the axis of rotation
  - Objects rotating will also have a radius associated with them
    - Any “r” that appears from here on out stands for radius
  - The topic of circular motion will bring back ALL of the topics that we have discussed previously
    - The equations are even very similar
    - All we are doing is changing the variables
    - Let’s start at the beginning of the course, with some definitions before we move on to kinematics and beyond

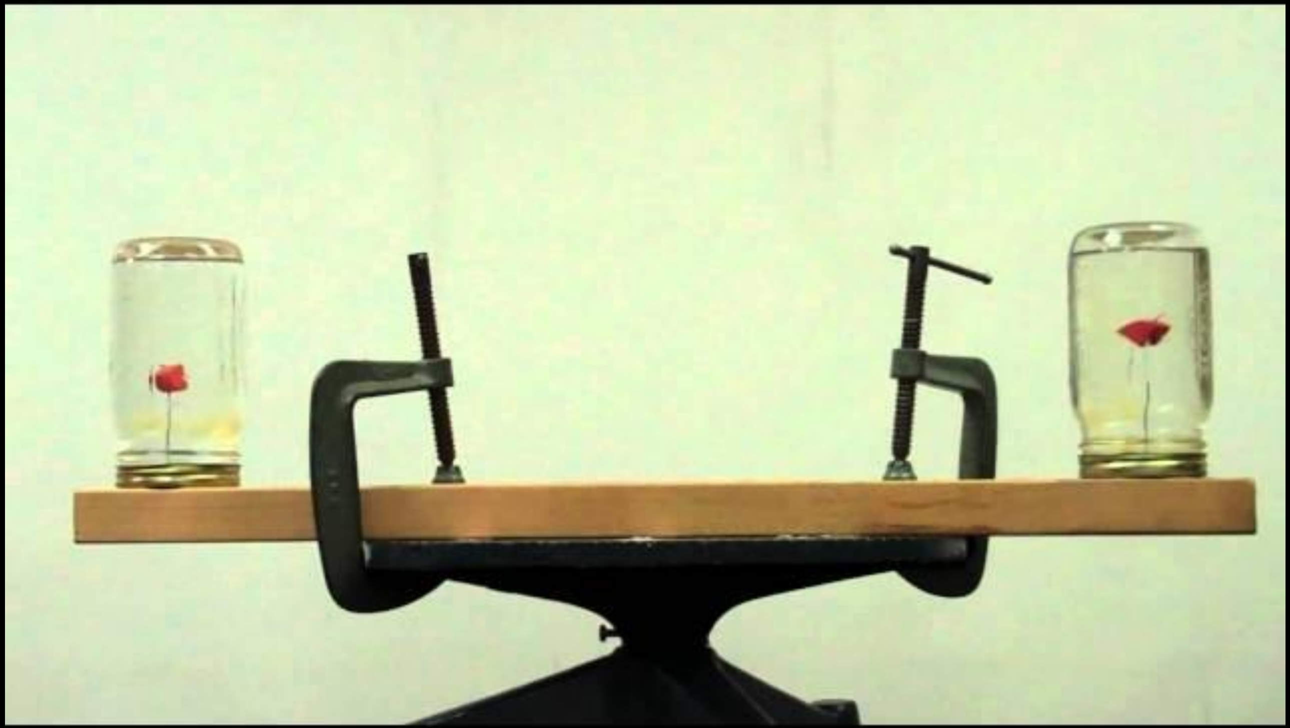
- Tangential Velocity
  - When an object rotates, it has a linear velocity that points in a line tangent to the circular path it is following
    - This is the tangential velocity
    - While not entirely accurate, it is helpful to think that this is the sideways velocity that the object is moving in a circle
  - Any object traveling in a circular path has a tangential velocity given by the following equation

$$\boxed{v_T = \frac{2\pi r}{t}}$$

  - Why does this equation make perfect sense?

- Centripetal Force and Acceleration
  - When an object rotates, it is constantly changing direction
    - If an object is constantly changing direction then the object must be accelerating
      - If there is an acceleration then there must also be a force
    - This force is called the centripetal force and therefore the acceleration is called the centripetal acceleration
  - The force vector (and therefore the acceleration) point towards the center of the circle
    - The word centripetal means “center seeking”

- Here are two videos as examples of centripetal force



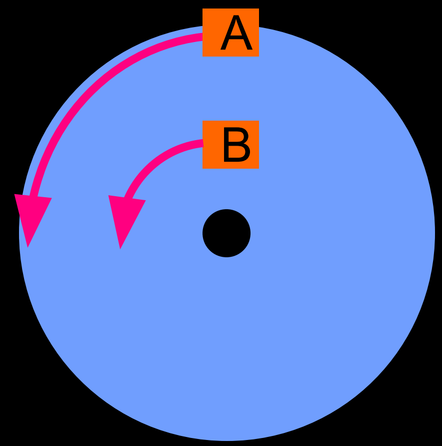
- Centrifugal Force
  - The centrifugal force, or “center-fleeing” force is a fictitious force
    - Instead, the centrifugal force is really a manifestation of inertia
    - In the first video, the water in the jars moves outward
      - This is NOT due to a force
      - This is due to the inertia of the water itself
      - Due to the inertia, the water wants to stay where it was, so it moves out to try to accomplish this
    - It is the same when you go around a turn too fast in a car and are thrown to the outside of the car
      - You move, not because of a force, but because of your inertia

# Rotational Kinematics

- Angular Displacement ($\theta$)
  - Angle swept out as a rigid body rotates around an axis of rotation
    - Units: Radians (rad)
  - By convention, the direction of angular vectors is found using the right-hand rule

- Direction of Angular Vectors
  - Angular displacement, angular velocity, and angular acceleration are all vectors
    - Given that they are vectors, they need to point in a direction
    - The issue is that these vectors are associated with something that is rotating
      - By its very definition, a rotating object changes its direction constantly
    - To get around this apparent problem, scientists have adopted the convention that the angular vectors point along the axis of rotation
      - The direction along the axis of rotation is determined by the Right-Hand Rule
        - » The Right-Hand Rule states that when the fingers of your right hand curl in the direction of the motion, your thumb points along the direction of the vector

– Let’s look at the Right-Hand Rule and the Ferris wheel




– If you curl the fingers of your right hand in the direction of motion, then your thumb points towards you

» In other words, it points out of the screen

» To draw a vector that points out of a flat surface we will use a dot with a circle around it, like $\odot$

• By convention, these are positive

» To draw a vector that points into a flat surface we will use a circle with an X within, like $\otimes$

• By convention, these are negative

- Angular Velocity (ω)
  - Analogous to linear velocity
  - How fast the object is rotating
    - Units: Radians/second (rad/s)
  - By definition, angular velocity is equal to the change in angular displacement over the time

$$\boxed{\omega = \frac{\Delta\theta}{t}}$$

- Angular Acceleration (α)
  - Analogous to linear acceleration
  - At what rate an object is changing its angular velocity
    - Units: radians per second per second
      - rad/$s^2$
    - By definition, angular acceleration is equal to the change in angular velocity over the time

$$\boxed{\alpha = \frac{\Delta\omega}{t}}$$

- Rotational Kinematics Equations
  - These equations are identical to the linear kinematics equations with the exception of what variables are present

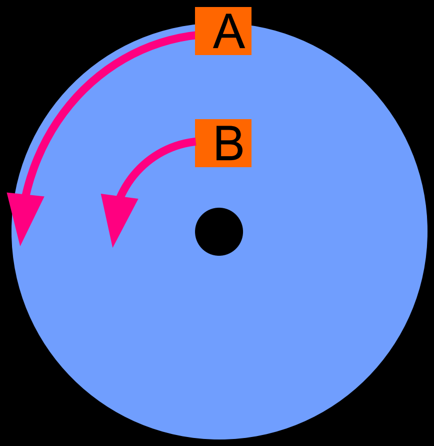
$$\boxed{\omega = \omega_o + \alpha t}$$

$$\boxed{\omega^2 = \omega_o^2 + 2\alpha\theta}$$

$$\boxed{\theta = \frac{1}{2}(\omega_o + \omega)t}$$

$$\boxed{\theta = \omega_o t + \frac{1}{2}\alpha t^2}$$

- Angular and Tangential Variables
  - Picture two spots on a Ferris wheel
    - Since they are on the same Ferris wheel, then it follows that they would have the same angular velocity
      - I.e. they are rotating at the same rate
    - Based on the picture below, which spot has a higher tangential velocity?



- Since Point A and Point B are at different radii from the axis of rotation, they will have different tangential velocities
- The relationship between tangential velocity and angular velocity is

$$\boxed{v_T = r\omega}$$

- The same relationship holds true for tangential acceleration and angular acceleration

$$\boxed{a_T = r\alpha}$$

- The same relationship holds true for tangential displacement and angular displacement
  - In fact, the following is the definition of arc length of a circle

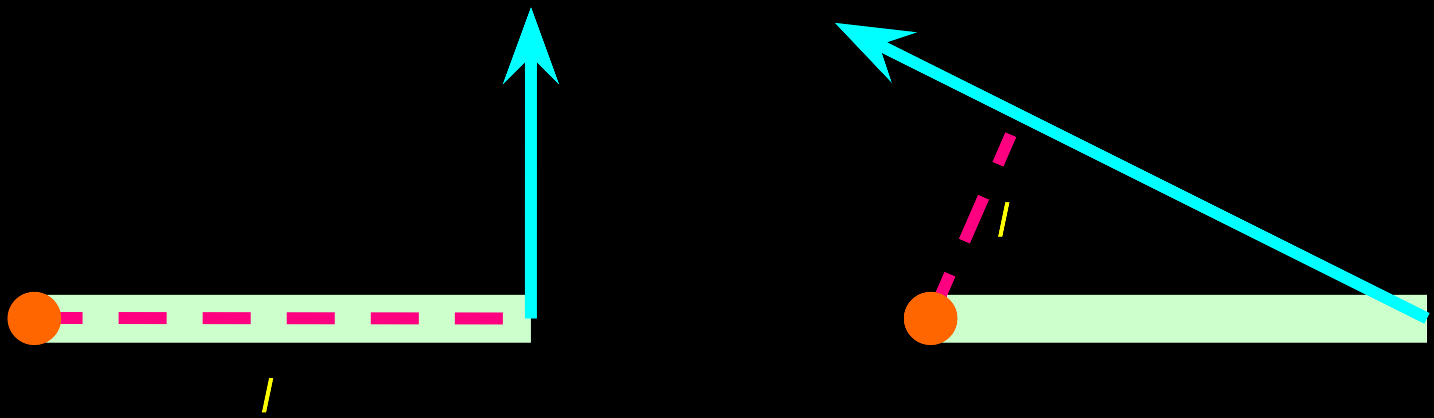
$$\boxed{x = r\theta}$$

- Torque ($\tau$)
  - Analogous to force
    - Forces cause linear accelerations
    - Torques cause angular accelerations
  - If a force acts on a rigid object and does not act through the axis of rotation, it creates a torque
    - Units: Newton·meter
    - Torques are just that, forces that are acting off of the axis of rotation and therefore cause rotations
    - Therefore, to mathematically describe a torque requires us to know two things
      - How large the force is
      - How far away from axis of rotation the force acts

$$\boxed{\tau = Fl}$$

*l* = lever arm

- The lever arm is the term used to describe how far away from the axis of rotation the force is
- The lever arm of the force is the straight line distance between the torque and the axis of rotation
  - It is always perpendicular to both




In both of the above pictures, the pink dashed line is the lever arm because it is the shortest distance between the torque and the axis of rotation

- Mass in Rotational Motion
  - When we have been talking about linear motion, we have been making the assumption that all of the mass is concentrated at the center of the object
    - At a point we called the center of mass
  - This assumption is flawed when it come to talking about rotational motion
    - Where the mass is, specifically how it is spread throughout the rotating object, plays a role in determining how the object will respond to torques
    - Because of this, it is more helpful to talk about the moment of inertia of an object instead of its mass
      - The moment of inertia describes not only the mass of the object but also takes into account where the mass is located relative to the axis of rotation
        - » Moment of Inertia (I)

- For a point mass, the moment of inertia = $mr^2$
  - The moment of inertia will vary based on the shape
- The moment of inertia of the object depends on its shape and the position of the rotational axis
  - Keep the information on the next slide handy, you will need it throughout the circular motion portion of the class

| Moments of Inertia | |
|---|---|
| Shape | Moment of Inertia (I) |
| Point Mass | $I = mr^2$ |
| Hollow Sphere | $I = \frac{2}{3}mr^2$ |
| Solid Sphere | $I = \frac{2}{5}mr^2$ |
| Thin Hoop rotating around its center | $I = mr^2$ |
| Solid Disk rotating around its center | $I = \frac{1}{2}mr^2$ |
| Thin Cylinder | $I = mr^2$ |
| Solid Cylinder | $I = \frac{1}{2}mr^2$ |
| Thin rod rotating around its end | $I = \frac{1}{3}mr^2$ |
| Thin rod rotating around its center | $I = \frac{1}{12}mr^2$ |

- Newton's Second Law for Rotation
  - Instead of forces we are looking at torques
    - Instead of mass we have the moment of inertia
    - Instead of linear acceleration we have angular acceleration
      - Other than that, the equation is the same

$$\boxed{\tau_{net} = I\alpha}$$

- Equilibrium
  - Equilibrium means no acceleration
    - Therefore the linear and angular acceleration both must be zero
    - Therefore, the net force in all directions AND the net torque must be ALL equal to zero
      - The same strategies that applied to solving equilibrium problems the first time apply here

$$\boxed{\Sigma F_x = 0}$$

$$\boxed{\Sigma F_y = 0}$$

$$\boxed{\sum \tau = 0}$$

- Angular Momentum (L)
  - Similar to linear momentum, angular momentum is the product of the moment of inertia and the angular velocity
    - Units: Newton·meter$^2$·radians/second
      - N·m$^2$·rad/s
  - The direction of the angular momentum follows the right-hand rule

$$\boxed{L = I\omega}$$

- Conservation of Angular Momentum
  - The total angular momentum of a system will be conserved if the net external torque is zero
  - Just like a top, objects that are spinning will resist a change in the direction of rotation
    - In the tops case, it will resist falling over

- Work in Rotational Motion
  - Similar to linear work, rotational work is defined as the product of the torque and the angular displacement

$$\boxed{W_R = \tau\theta}$$

- Rotational Kinetic Energy
  - The rotational kinetic energy of an object is defined as

$$\boxed{K_R = \frac{1}{2} I\omega^2}$$

  - Now that we have a definition for rotational kinetic energy, then everything we did with conservation of energy can apply with this new term