# **Two-Dimensional Motion**

By the end of this unit, you will be able to:

Explain why a 2D kinematics problem can be modeled as two 1D kinematics problem

Solve a 2D kinematics problem

Solve a projectile problem when the object is launched with an initial velocity at an angle above the ground

Solve a projectile problem when the object is launched with a horizontal initial velocity

- Two-Dimensional Motion
  - Two-dimensional motion is motion in two directions
    - Typically along the x and y directions
  - Kinematics equations apply to 2D motion as well as linear motion
    - As long as the acceleration is constant
      - If you remember, that is a requirement to use kinematics
  - When an object is moving, the motion along any axis is independent of all other axes.
    - The x and y axis are independent of each other
    - If they are independent then they do not affect/influence/impact the other direction
      - Say I have an object moving in the x and y directions
      - If I suddenly stop all motion in the y direction, the object will continue to move in the x direction since the y direction does not affect the x direction

- Knowing this, all 2D motion problems can be simplified to two linear motion problems
- Essentially, this is all there is to know about two-dimensional motion

- Projectile Motion
  - Projectile motion is the curved motion of an object that is given an initial velocity and then moves as though in free fall
    - What is true about an object in free fall?
  - Every projectile has a trajectory
    - The trajectory is just the curved path followed by a projectile
    - The shape of this path will always be a parabola
  - Projectile motion is a specific type of 2D motion
    - The projectile is moving in the x and y directions
    - So, like all 2D motion, the two dimensions are independent of each other
    - Meaning that I can solve a projectile problem be looking at two linear motion problems

### - Acceleration

- Since the object is in free fall, the only thing acting on the object is gravity
- Gravity only acts in the y-direction
  - The acceleration in the y-direction is  $9.8 \text{ m/s}^2$
  - I will use the symbol  $a_v$  to reference the acceleration in the y-direction
- There is no acceleration in the x-direction
  - So,  $a_x = 0$
  - What does this mean about the velocity in the x-direction?

$$a_x = 0$$
  $a_y = g = 9.8m/s^2$ 

Initial Velocity

- Projectile starts with an initial velocity that can be broken up into components.
  - $-v_{ox}$  and  $v_{oy}$  are the components of the given initial velocity  $v_{ox}$
- Vertical Velocity at the Top of the Trajectory
  - The speed in the y-direction at the top of the trajectory is always equal to zero
  - Why is this the case?
  - Depending on the problem, this could be  $v_{ov}$  or  $v_{v}$ 
    - You have to determine which from the problem

## - Final Velocity in the X-Direction

- Since there is no acceleration in the x-direction, the velocity in the x-direction is constant
  - No acceleration means no change in the velocity
  - This means that  $v_x$  is always equal to  $v_{ox}$



- How long something is in the air (hang time) is only determined by the height of the object
- Since the height is in the y-direction, only the y-direction determines the time
  - As we work projectile problems, finding time from the y-direction (if possible) is a great place to start
- Time is also the only quantity that links the x and y-directions together
  - Because of this, finding time is a wise thing to do first
- Range
  - The range is how far the projectile will travel in the x-direction









Let's use this problem as an example



#### He has a picture, which is great



He has some of his given information written down

Let's finish it off

When writing out your given for a projectile problem, it is helpful to organize it like I have below



How do I go about finding the initial velocities?



Resolve the given initial velocity into components

x = y =  $v_{ox} = 86.6 \text{ m/s}$   $v_{oy} = 50 \text{ m/s}$   $v_x = v_y = 0 \text{ m/s}$   $a_x = 0 \text{ m/s}^2$   $a_y = -9.8 \text{ m/s}^2$  t = $t_{total} = ?$ 

Which direction (x or y) do I have enough information to solve for t?



x = y =  

$$v_{ox} = 86.6 \text{ m/s}$$
  $v_{oy} = 50 \text{ m/s}$   
 $v_x = v_y = 0 \text{ m/s}$   
 $a_x = 0 \text{ m/s}^2$   $a_y = -9.8 \text{ m/s}^2$   
 $t =$   
 $t_{total} = ?$ 

#### Using the y-direction, I can solve for t

$$v = v_o + at$$

$$t = \frac{-v_o}{a}$$

$$t = \frac{-50}{-9.8}$$

$$t = 5.1s$$

Is this the total time in the air?



x = y =  

$$v_{ox} = 86.6 \text{ m/s}$$
  $v_{oy} = 50 \text{ m/s}$   
 $v_x = v_y = 0 \text{ m/s}$   
 $a_x = 0 \text{ m/s}^2$   $a_y = -9.8 \text{ m/s}^2$   
 $t = 5.1 \text{ s}$   
 $t_{total} = ?$ 

## To find the total time in the air I need to double the time to the top

$$t_{total} = 10.2s$$

Let's see how Peter did...





