Two-Dimensional Motion

By the end of this unit, you will be able to:

Explain why a 2D kinematics problem can be modeled as two 1D kinematics problem
Solve a 2D kinematics problem
Solve a projectile problem when the object is launched with an initial velocity at an angle above the ground
Solve a projectile problem when the object is launched with a horizontal initial velocity

- Two-Dimensional Motion
- Two-dimensional motion is motion in two directions
- Typically along the $x$ and $y$ directions
- Kinematics equations apply to 2D motion as well as linear motion
- As long as the acceleration is constant
- If you remember, that is a requirement to use kinematics
- When an object is moving, the motion along any axis is independent of all other axes.
- The $x$ and $y$ axis are independent of each other
- If they are independent then they do not affect/influence/impact the other direction
- Say I have an object moving in the $x$ and $y$ directions
- If I suddenly stop all motion in the $y$ direction, the object will continue to move in the x direction since the y direction does not affect the x direction
- Knowing this, all 2D motion problems can be simplified to two linear motion problems
- Essentially, this is all there is to know about two-dimensional motion
- Projectile Motion
- Projectile motion is the curved motion of an object that is given an initial velocity and then moves as though in free fall
- What is true about an object in free fall?
- Every projectile has a trajectory
- The trajectory is just the curved path followed by a projectile
- The shape of this path will always be a parabola
- Projectile motion is a specific type of 2D motion
- The projectile is moving in the $x$ and $y$ directions
- So, like all 2D motion, the two dimensions are independent of each other
- Meaning that I can solve a projectile problem be looking at two linear motion problems


## - Acceleration

- Since the object is in free fall, the only thing acting on the object is gravity
- Gravity only acts in the y-direction
- The acceleration in the $y$-direction is $9.8 \mathrm{~m} / \mathrm{s}^{2}$
- I will use the symbol ay to reference the acceleration in the $y$-direction
- There is no acceleration in the $x$-direction
- So, $a_{x}=0$
- What does this mean about the velocity in the x-direction?



## - Initial Velocity

- Projectile starts with an initial velocity that can be broken up into components.
$-v_{o x}$ and $v_{o y}$ are the components of the given initial velocity $v_{o}$


## - Vertical Velocity at the Top of the Trajectory

- The speed in the $y$-direction at the top of the trajectory is always equal to zero
- Why is this the case?
- Depending on the problem, this could be $\mathrm{v}_{\mathrm{oy}}$ or $\mathrm{v}_{\mathrm{y}}$
- You have to determine which from the problem
- Final Velocity in the X-Direction
- Since there is no acceleration in the x-direction, the velocity in the $x$-direction is constant
- No acceleration means no change in the velocity
- This means that $v_{x}$ is always equal to $v_{o x}$


## - Time

- How long something is in the air (hang time) is only determined by the height of the object
- Since the height is in the $y$-direction, only the $y$-direction determines the time
- As we work projectile problems, finding time from the $y$-direction (if possible) is a great place to start
- Time is also the only quantity that links the x and y -directions together
- Because of this, finding time is a wise thing to do first
- Range
- The range is how far the projectile will travel in the x-direction




Let's use this problem as an example


He has a picture, which is great


He has some of his given information written down

Let's finish it off

When writing out your given for a projectile problem, it is helpful to organize it like I have below

$$
\begin{array}{ll}
x= & y= \\
v_{o x}=\text { Find } & v_{\text {oy }}=\text { Find } \\
v_{x}= & v_{y}=0 \mathrm{~m} / \mathrm{s} \\
a_{x}=0 \mathrm{~m} / \mathrm{s}^{2} & a_{y}=-9.8 \mathrm{~m} / \mathrm{s}^{2} \\
\quad t= & \\
\quad t_{\text {total }}=? &
\end{array}
$$



Resolve the given initial velocity into components

$$
\begin{array}{ll}
x= & y= \\
v_{o x}=86.6 \mathrm{~m} / \mathrm{s} & v_{o y}=50 \mathrm{~m} / \mathrm{s} \\
v_{x}= & v_{y}=0 \mathrm{~m} / \mathrm{s} \\
a_{x}=0 \mathrm{~m} / \mathrm{s}^{2} & a_{y}=-9.8 \mathrm{~m} / \mathrm{s}^{2} \\
\quad \mathrm{t}= & \\
t_{\text {total }}=? &
\end{array}
$$

Which direction (x or y) do I have enough information to solve for t?


\[

\]

Using the $y$-direction, I can solve for $t$

$$
v=v_{o}+a t
$$



$$
t=5.1 s
$$



$$
\begin{array}{ll}
x= & y= \\
v_{o x}=86.6 \mathrm{~m} / \mathrm{s} & v_{o y}=50 \mathrm{~m} / \mathrm{s} \\
v_{x}= & v_{y}=0 \mathrm{~m} / \mathrm{s} \\
a_{x}=0 \mathrm{~m} / \mathrm{s}^{2} & a_{y}=-9.8 \mathrm{~m} / \mathrm{s}^{2} \\
\qquad & t=5.1 \mathrm{~s} \\
t_{\text {total }}=?
\end{array}
$$

To find the total time in the air I need to double the time to the top

$$
t_{\text {total }}=10.2 \mathrm{~s}
$$

Let's see how Peter did...




